The Derivative of $|x|$

The slope of the graph of $f(x) = |x|$ changes abruptly when $x = 0$. Does this function have a derivative? If so, what is it? If not, why not?

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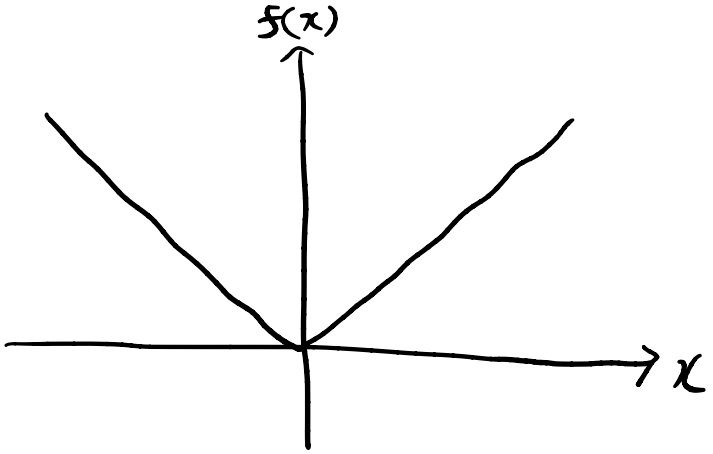
8/7/25

$$f(x) = |x|$$



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

At $x = 0$, $f'(0) = \lim_{\Delta x \to 0} \dfrac{f(0+\Delta x) - f(0)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{f(\Delta x)}{\Delta x}$$

From the right, $\lim_{\Delta x \to 0^+} \dfrac{\Delta x}{\Delta x} = 1$

From the left, $\lim_{\Delta x \to 0^-} \dfrac{-\Delta x}{\Delta x} = -1$

The limit as $\Delta x \to 0$ does not converge to a single value.

$\therefore$ $f'(0)$ is undefined. $f(x)$ is not differentiable at $x = 0$.

$\therefore$ $f(x)$ is not a differentiable function.